Extract from Syllabus for Paper 2, Topic1 : The candidate should make a study of the concept and development of management as science and art drawing upon the contributions of leading thinkers of management and apply the concepts to the real life of government and business decision making keeping in view the changes in the strategic and operative environment.

1. Quantitative Techniques in Decision Making:

- 1.1 Descriptive statistics – tabular, graphical and numerical methods,
- 1.2 Introduction to probability, discrete and continuous probability distributions,
- 1.3 Inferential statistics: sampling distributions, central limit theorem, hypothesis testing for differences between means and proportions, inference about population variances,
- 1.4 Chi square and ANOVA,
- 1.5 Simple correlation and regression, time series and forecasting, decision theory, index numbers;
- 1.6 Linear programming – problem formulation, simplex method and graphical solution, sensitivity analysis.

Note: Understanding the concept to apply them in real life situations is the key. The contents in this pdf do not reproduce what is already available in text books on the subject. The contents are the gist of fundamental points on the topics that can be easily understood and applied to solve problems by learners. A few worked out examples are given for the more complex concepts of regression and chi square, linear and non-linear.

Along with this pdf review the basic concepts in NCERT Mathematics text books from class 8 to 12 that are also available in the School Unit.

Source : Adapted from Lecture Notes of M. Phil (Law and Governance) and Self- study Material for MS 8 and MS 95 Research Methodology for Management Decisions, for MBA (Finance) IGNOU, and other readings.

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LEARNING MATERIAL

Introduction

PURPOSE: Quantitative Decision Making is a complex managerial activity. The techniques are the tools that need to be understood. Their application areas are to be known. Computers use quantitative techniques for real life problems to suggest an optimal solution.

1. Quantitative techniques facilitate management to become efficient and creative. They enable a manager to demonstrate, and defend the decision made and to negotiate for funds.

FIGURE 1

1. Statistical Techniques (ST)

- Involve Statistical Data = Numerical representation of quantitative aspects of resources / problem that has been collected as per statistical methods.

  Data can be a) Primary or b) Secondary
- Statistical Methods are tools or devices of analysis and synthesis , for systematically collecting collecting, organising classifying information, for interpreting.

2. Operation Research (OR) Techniques

- In ST and OR, a problem is to be defined, and solved in a scientific manner, based on quantitative tools and methods, to give optimal solution, in the form of a model that represents the reality of the solution.
- Characteristics of Data : It must aggregate the facts, be affected by multiplicity of causes; be a reasonable estimate; collected in a systematic manner; numerically expressed, placed in relation to each other.

Chapter 1 Descriptive Statistics

1. Descriptive statistics: tabular, graphical and numerical methods of presentation of data.

1.1 Statistical data is a set of facts expressed in quantitative form. It is expressed in measurable quantities such that decision making becomes easier. Descriptive statistics deals with arranging and presenting the data that has been collected in primary or secondary form. Tabular, graphical and numerical methods enable the data to be condensed, make it comparative, and highlight the significant features at a glance.

1.2 Classification is numerical method and may be quantitative or qualitative.

a) Data may be classified as classes or frequencies. Distinct group of raw data is called classes. Number of classes should not be too few or too many.

b) The number of observations made in each class is called frequency. When the variable in the data is continuous, it is called a continuous data frequency and when not continuous it is called discrete data frequency. An example continuous frequency data is Age in years 20 to 25, 25 to 30, 30 to 35. 35 to 40 and the number of workers in each age group are 15, 22, 38, and 47, respectively. Discrete data frequency would be number of employees who are 20 years or more, 30 years or more, 40 years or more etc.

c) Each class has a lower and an upper limit, such as less than 60 and more than 69. This means that the upper limit of preceding class becomes the lower limit of succeeding class.

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d) Class interval represents the span or size. Class interval can be Exclusive or Inclusive or Open ended. In Exclusive interval the upper limit of one class is lower limit of the next class. In Inclusive interval the upper limit of one class is included in that class itself. Open ended distribution the lower limit of the first class and the upper limit of the last class is not given. Example, the first class will state Less than 1000, 1,000 to 2000… 25,000 and above.

1.3 Graphical method: graphs enable a quick representation of tabular and numerical data. It represents the frequency of the data, and can be presented in the form of a Bar Diagram (separate bars for each time span) , Histogram (Bars adjacent of one another) , Frequency Polygon (is a combination of bars and frequency curve) , Ogive or Cumulative Frequency Curve can be Less than Ogive or More than Ogive.

Introductory:

Terms used in descriptive statistics.

a) Averages: Concepts must be very clear.
b) Sampling for descriptive data, because to know the complete is impossible. Kafka methodology of writing called 'metamorphosis' is to be adopted for sampling.
   Borges (Argentinian) addresses the problem of knowledge and has deeply influenced post modernism.
   Just as a story discussing maps for topography, miniaturizes the land mass to communicate knowledge, similarly, sampling tried to convey about existing reality but sampling is not complete reality.
   The bigger the sample, the more accurate is the probability because the sample becomes a perfect map, coterminous with the terrain itself. It is absurd to argue that sampling is not acceptable

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because to know an object its entirety must be known. If this were true then nothing in this world or the universe could be known.

c) Standard Error: must be well understood.
d) Inference: best use in differentiating between same or similar products, such as differentiation of variety of Tea. Tasting Tea)
e) Probability: best understood through Philosopher Ian Hacking's perspective.

Chapter 2

1.1.3. Numerical methods: Averages:

1.1 Many kinds of estimators that give the value of parameter are:

a) Simple arithmetic mean: x = sigma x_i divided by n
b) Sample median = middle value
c) Sample Harmonic Mean is the inverse of arithmetic mean: 1 divided by sigma x_i divided by n.
d) Geometric mean ($x_{i\ \text{multiplied}}$ with each other) x_n) to the power of 1/n.
e) Distribution represented by sigma with i = 1= (x_i – x divided by n) ^{n} variance in the population deviation.
f) (s^2) = Standard deviation of a sample = sigma with i=1 (xi – arithmetic mean) ^{2} divided by n. It shows how much does each unit of data deviate from the Centre. For finding how far is the sample element from the Centre, the same formula is used after replacing arithmetic mean with '$/^u$'.
g) Estimators of variance and estimators of mean come up with value.

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h) The Renaissance movement 17th century laid the foundations of modern thinking and life based on 'chance'. Sampling and Probability show the role of chance in existence.

1.2 Average and Scatter [i] : The purpose of average is to represent a group of individual values in a simple and concise manner. Average acts as a representative number and can be misleading. Yet it is easy to grasp and therefore, is widely used as a general assessment. The measure of central tendency called Median, Mode and Mean or Average are Positions of the scatter.

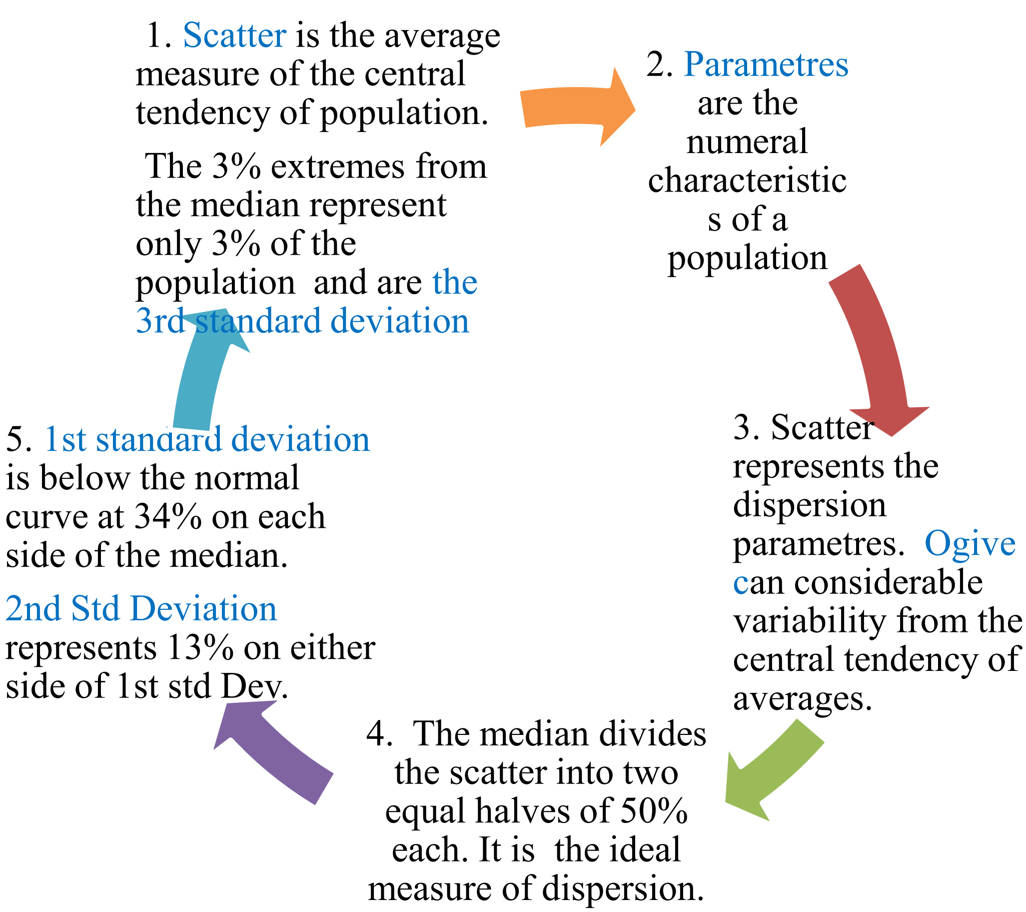

FIGURE 2

1.1.4. MEASURE OF CENTRAL TENDENCY [ii] :

1.1.4.1. Several kinds of averages and scatter exist.

a) In a perfectly symmetrical distribution, arithmetic mean, median and mode coincide in a single line.

b) In a moderately skewed distribution, Mean minus Mode = 3 (mean – median) Asymmetrical scatter or skew is positive skew or negative skew. The area of the scatter (also called the 'deviation') gets divided in proportion to the frequency called 1st Standard deviation, 2nd standard deviation, and 3rd standard deviation. See Figure 3 on page 13.

1.1.4.2. Mathematical averages include:

a) Arithmetic mean (AM) is the centre of a distribution, but can be incorrect. For example, if an airplane flies as four different speeds of 100 + 200+ 300+ 400 / 4 = 1000 / 4 = 250 km. per hour is NOT the correct average velocity. This is because with each change in speed, time taken varies.

This example with HM gives 1 hour + 30 minutes + 20 minutes + 15 min = 2hrs 05 minutes = 25 /12 hours. Average velocity Reciprocal = 400 /1 divided by 25/12 = 192 M.P.H. (HM)

a) Arithmetic mean: For ungrouped data: Sigma f x divided by sigma f n. Here, f is frequency, 'x' is central value of class interval and n is total numbers. Also, A + sigma d divided by n. d = x minus A. Here A is the assumed mean.

b) Arithmetic mean for discrete and continuous series:

a) Sigma f x over sigma f. Sigma sign is similar to W standing upright with its sides aligned to 12 and 6 on a clock , instead of 9 and 3 for W. It means summing up or total. Small sigma is 'o' with a tiny line growth on top.

b) Also A + sigma f d divided by sigma f.

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Here d = x minus A.

b) Harmonic mean. (HM) is reciprocal of arithmetic mean. It is appropriate for rates and prices, and also where variables are involved. It can add up a sting of 'items' to make up the set whose average is required.

Sigma = summing up or totaling. Small sigma is 'o' with a tiny line growth on top.

In Harmonic Mean: average rate of increase of variate is calculated as

a) Ungrouped data: HM = n divided by $1/x_1 + 1/x_2 + \ldots + 1/x_n$

b) Discrete series and continuous series: HM = sigma f / sigma (f/x)

c) In Algebraic notations letters stand for numbers as per stated value. This is a very compact method for calculation. By applying an algebraic formula, the problem is solved once and for all. The formula is the answer. Substitute the actual quantities for the letters in the formula to get the solution.

For example, the formula for Arithmetic mean is $x_1+x_2+x_3+x_4$ divided by n. Here x are the different numbers and 'n' is the total numbers added. In the above example four different speeds of the airplane are from x_1 to x_4 and 'n' is 4. The short formula for AM = 'sigma x / n'.

The formula for HM or (H) = n / sigma (1/ x)

Same example as above is written in this formula as

4 / (1/100 + 1/200 + 1/300+ 1/400)

= 4 / (25/1200) = 4 x1200 / 25 = 192 m.p.h.

d) Geometric mean (G): N under root $x_1+x_2+x_3+x_4$ = G

For calculation of Geometrical Mean: average rate of increase of variate.

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a) Ungrouped data: GM = (x1 minus x2 into x3 ….. x_n)1/n of GM = i/n sigma 4x. Here x_c is the mid value of the class interval.

b) Discrete series and continuous series, GM = sigma f lgx / sigma f where f = corrected frequency of the variable x of variate.

Geometrical Mean passes through the centre of gravity of the distribution or the scatter. The mean is the abscissa of the centre of gravity.

e) MODE is the value of variate which occurs most frequently in an ungrouped data. For continuous series: Mode = L + (f $_0$ minus f_0 divided by 2 f $_1$ minus f $_0$ minus f $_2$) into small i where L is the lower limit of class interval in which mode is situated; f $_1$ is the maximum frequency; f_0 is frequency of the preceding class interval, f_2 is the frequency of succeeding class interval; i is the length of the class interval.

f) When distribution is not normal: Mode = 3 median minus 2 mean (mean minus mode = 3 (mean – median)

AM > GM > HM

(AM) (HM) = (GM) 2

h) MEDIAN: which is also the positional average is the middle term of the array, when the ungrouped data is arranged in ascending or descending order.

For ungrouped data:

Median is size (u+1/2) item if n is an odd number. If n is an even number, then median = [size of n/2 item + f (n/2 +1) item] divided by 2.

For continuous series

Median = L+ small i / f (n/2 minus c) where

n = total frequency;

L = lower limit of grade in which median number lies;

i = class interval;

f = frequency of the median group and

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c = the cumulative frequency of the group preceding the median group.

1.1.4.1. Partition values: If the division of a series is into

a) 4 equal parts, each part is called Quartiles. The first Q1 = n/4; Q3 is 3n/4. For grouped data: Q1 = l + small I /f (n/4 minus c).
b) 10 equal parts, each part is called deciles;
c) 100 equal parts, each part is called percentiles.

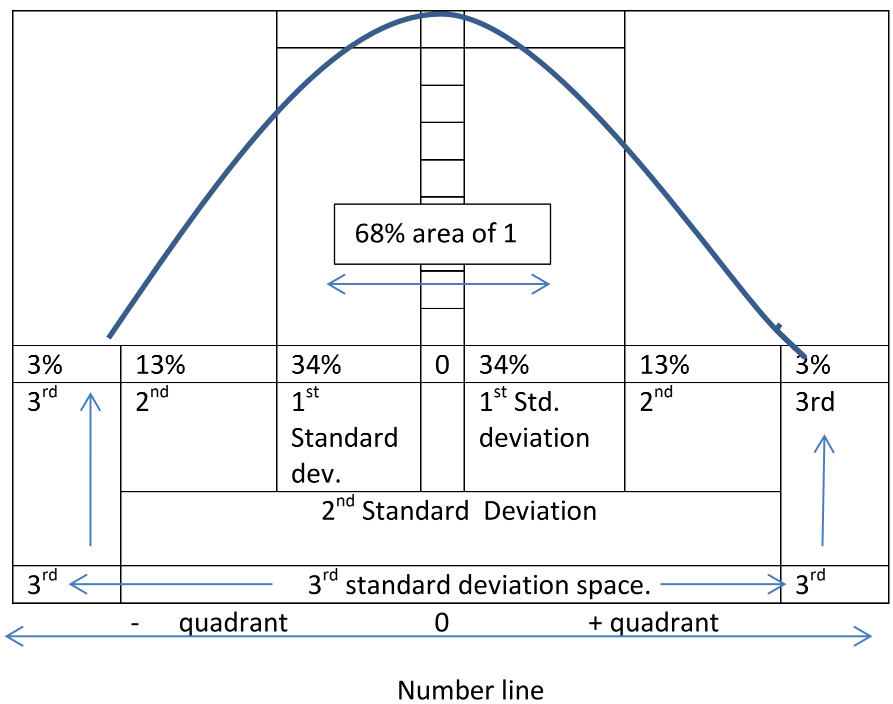
a) Distribution when symmetrical is about their central value. = the Normal curve or the Bell curve.
Mode Mean and Median are the same in a perfect distribution.
They divide the area into two equal halves of = 50% each side of 0.
This 50% has three sub divisions that equal D_1= 34% area; D_2 13% area and D_3 = 3% area. (34 + 13 + 3 = 50) These are the three standard deviations from the mean that represent the error in sampling.

b) Formula for calculation of Mean deviation = sigma /x-x dash above / divided by n. Here x dash above = 100 and S =13
The graph given on next page helps to visualize the placement of standard deviation errors in a Sample.
1. 0 is the point at which the Mean, Mode and Median coincide in a perfect sample. This makes the sample 100 % correct. The space shown at 0 is indicative only, and does not exist in a perfect sample.
2. In the Error of first standard deviation, the three averages of mode, mean, median could be located anywhere in the 34% area, both on the positive Quadrant and the negative Quadrant on the number line.
3. The Error of 2nd standard deviation would be at any point in the 13% space.

4. The Error of 3rd standard deviation would be at any point in the 3% space on both extremes of the curve.

c) In a graph the class boundary or parameters are on horizontal base called x and frequency is on vertical base called y. The frequency of the entire area = 1.

FIGURE 3



In a normal curve there are two parametres called x and y.

d) 1st Parameter is x the value of which is given. Its placement in the quadrant of the number line depends on the given value of x.

e) 2nd Parameter is y that represents the frequency. Its position on the Vertical axis will be the straight line from the given value of x.

f) The population of the parametres is represented by 'sigma'.

g) Any factor or element that is distributed normally can be converted into a standard distribution as per formula ' x – mew divided by

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sigma = z inverted U (for normal distribution) N (0,1) Readymade distribution tables are available of (o,1)

7.1 Relation between frequency, variation and parametre.

7.1.1. 68% observations fall between the 1st standard deviation on either side of 0 that represents the perfect coinciding of the mode, mean and median.

7.1.2. Probability of falling between sigma and mew is .68.

7.1.3. The sigma of mew is 68%.

7.1.4. Variance shows the spread out when the three averages are not coinciding. 2nd standard deviation is 95% and 3rd deviation is 99%.

7.1.5. Other distributions are derived from the Normal Distribution. Conditional Probability.

Chapter 3

Topic 1.3 Inferential Statistics: Sampling

1. Sampling is a statistical technique to know about the whole bulk (called a ‘ universe’) through study of a small part called the Sample. A sample is a part of the whole that is used to identify the characteristics of the whole. When rice is cooked, a sample of few grains is sufficient to know if the whole pot of rice is fully cooked or not. Similarly, the whole or the bulk is called the estimator of the characteristic. It is a function of sample values. Function here means the algebraic function of a + b = c.

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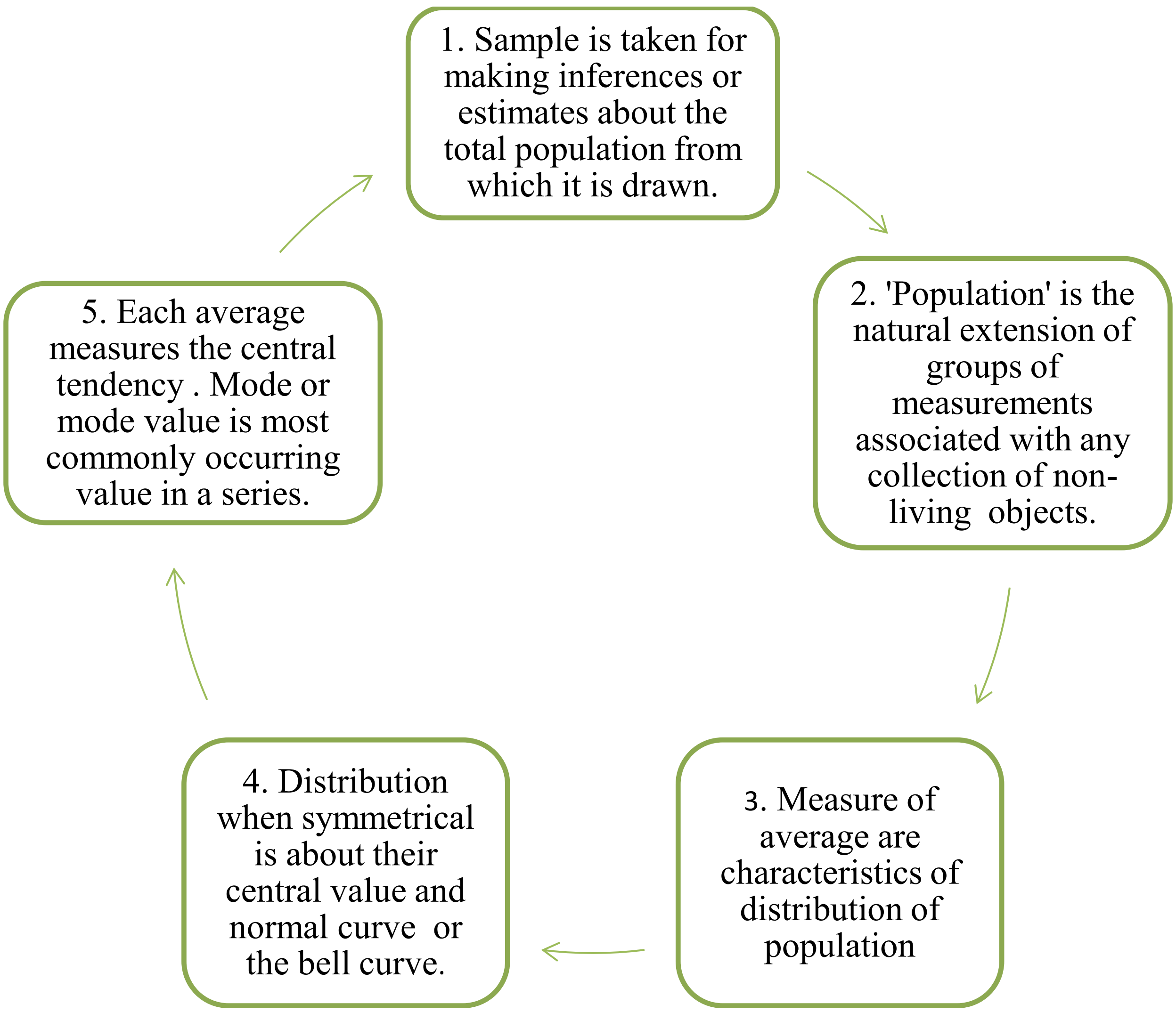
2. Sampling technique is for study of 'descriptive data' or 'qualitative data'. It is therefore, useful for research in Social Sciences. Sampling is important because to know the whole, through the study of the complete universe is not possible. If the entirety of a universe should be known, 'to know thing', or its characteristics, nothing can ever be known.

FIGURE 4 ON TERMS USED IN SAMPLING (Key concepts for understanding averages and deviations.)



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a. How to obtain a representative or a perfect sample?

1.3.1. Repeated sampling on basis of random choice gets a representative sample and will give estimates that differ and lead on to a sampling distribution.

1.3.2. Varying sample sizes. As size of the sample is increased, structure of the frequency begins to appear in the form of clustering of data frequency. When sample size is extremely large true value is obtained as the distribution of frequency fixes on to one point. As 'n' increases 'x' becomes closer and closer.

1.3.3. To collect a perfect sample, choose one based on 'pure chance' only. It should be a completely random sample. This alone will minimize the error. Larger the sample, lower the error. However, Research in Social Sciences is based on small sample sizes. This leads to error when converting data into graphs and charts. The Estimator will have a sampling distribution such as arithmetic mean.

1.3.4. Use of Chance and randomness of sample result in error from reality. This is notionally measured as the difference between reality and sample data. 'Unbiased' sample is representative of the whole. The biases that are built into a sample can be improved through stratified sampling. For stratified sampling the sample in divided horizontally into layers or strata, and equal number of samples are drawn from each of the strata.

1.3.5. Can error be formalized? How to capture this error? Can it be reduced?

1.3.6. When Sample is used to know the whole it leads to the zone of error. Inductive logic of 'small to big' is used in sampling. It is always subject to risk of Error. Problem inherent in data comes from sample error as relative of reality. Inductive logic risk is related to probability that cannot be predicted.

1.3.7. Is it representative of the whole is the key question for all data?

a) Population of the sample = all objects under study.

b) Characteristics of this population are the Mean of the population and Standard deviation.

c) Sample is part of the bulk used to identify the characteristic of the bulk. Bulk is called the 'estimator' (/^{u} read as new) of the characteristic. It is a function of sample values. Estimator is also referred to as 'the statistic'.

d) Sampling Error = error of the statistic or the error of the estimator = value of estimator minus True value.

e) If there is no difference in the sampling error, the sample represents the whole. If there is a difference, then the error exists.

1.3.8. For giving reliable results, a sample needs to be sufficiently large and representative of the universe. Such a sample is known to give a complete map of the characteristics of the whole. The key question for the reliability of a sample is 'whether or not it is representative of the whole?' As sampling technique gives only an 'estimated average', it is important to know the error in the sample.

1.3.9. The realm of the error is the problem inherent in the data. Inductive logic is used here for estimating the 'big' from the 'small'. Inductive reasoning is always subject to risk, because the sample may not be a representative sample. The question then is whether the error in the sample can be 'captured' and 'formalized'? Could it be reduced?

1.3.10. Randomness is the critical quality needed for any sample to be representative. How representative is the sample is the key question. It is assumed that the data has been selected randomly. But if it is biased and not random, the margin of error will be large.

1.3.11. Sample treated as a population for purpose of modelling. Probability is the proportion or the ratio.

a) P (A= event occurred) = 1-P (because probability is always located between 0 and 1). Then disjoint events are added to obtain the set of mutually exclusive events.

b) For example, in probability data on marital status married will be 1-P. Those never married or widow / widower or divorced are disjoint events to being married. 'Others' category could be included in the model to cover those spouses who have 'gone missing', have been abandoned without information.

c) Assigning probability to certain events through categories of phenomenon. The probability of just one event occurring does not exist. For example P = 1 is wrong. At least two events are needed for calculation of probability.

1.3.12. STRATIFIED SAMPLING:

a) A number between 0 and 1such as Sample space = set of all numbers between zero and one. On the number line S = {0 and I = infinity}. The probability of getting all elements in sample space = P = 1.

b) Between 0 and 1 the whole square on the graph is the Sample space. To find P (.3 and .7) a random variable (y) is needed. = (.3< or equal to y< .7). A density curve assigning a probability to every phenomenon is needed. Example .4x1=.4 (meaning 40% chance of getting a number between .3 and .7 is uniformly distributed in the sample space. This equal chance also exists for any other random number.

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Chapter 4 Probability, Logic and Language of probability

1.4. PROBABILITY

1.4.1. Probability cannot be predicted. Inductive logic risk is therefore inbuilt into all decisions based on probability.

In the simplest example of tossing of a coin, there are always two probabilities In the first toss the probability is 1, then as the number of tosses increase, the probability settles to 50% level. Therefore, the probability of an event occurring is basically a ratio or proportion of the total possibilities. (50: 50) This means half the time the coin will fall on its head and the other half times it will be tails, as the number of tosses increases. In a 40,600 times toss, this probability will hold, yet this in fact is an empirical (theoretical) result. It cannot be taken to represent reality of the whole.

1.4.2. LOGIC OF PROBABILITY:

a) The process involves the creation of a deductive set of reasoning. Rule of Probability / axioms are as follows:
b) P (A) satisfies O = P(A) = 1, meaning that probability lies between zero and one.
c) If 'S' is the sample space then probability of P(S) =1.
d) P (A does not occur) = 1 minus P(A)
e) Two events are considered disjoint or mutually exclusive if they have no outcomes in common, meaning that they cannot occur simultaneously. Disjoint probability = P (A and B) = P (A) + P(B).

1.4.3. Language of Probability :

a) Sample space is the set of all possible outcomes. S = (set of all possible values of x). Sample space S = {H, T,} in tossing of a coin.
b) Event is an outcome or a set of outcome of a random phenomenon. Therefore, an event is a subset of sample space and a random phenomenon.
c) Probability model is a mathematical structure that defines sample space, probability to events (to be assigned for calculation, meaning something will occur. What is the probability of its occurring?)

1.4.4. Major kinds of Probability Distributions :

a) Norman distribution / Gauss distribution for relationships between diverse phenomenon such as income, heights etc. Other distributions are derived from the Normal Distribution.
b) Basian Probability distribution: given one characteristic what is the probability of the other event?
c) Quasi distribution
d) Exponential Distribution

1.4.5. Basian Probability: Given one characteristic what is the probability of the other occurring:

a) Take a random probability number in one class. Total firearms 18940 divided by total females 31,510. What is the ratio of total females to total of all?
b) P (Total females as per random sample = 31, 510 = .193) / Total firearms = 18,940. Probability of females with firearm = 2559 divided by 6095 = .42
c) When this is represented as a Venn diagram:
    1. The entire square represents all the sample space (S)
    2. The two circles that do not touch each other represent characteristics A and B that are exclusive or disjoint.

3. When not disjoint, the circles intersect each other and have a common space between them. In this overlap, subtract the common area.
4. P (A or B) = P (A) + P (B) – (P (A and B)) In the last bracket (A and B) indicate that both are occurring. If both events are wanted, multiply both. P (A) x P (B). Most probability models assume that all events are independent and can be multiplied to find the whole.
   Denominator = the outside remaining.
   Numerator = the sample taken. In this sample it is assured that both are independent events.
5. Event A and B P (A+B) = P (A) x P(B divided by A) Both events = one event occurring x given first event occurring , then second event must occur.
6. When P (A) > 0 .
7. Conditional Probability P (B divided by A) = P(A+B) divided by P(A)

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Chapter 5 Linear programming

Problem formulation, simplex method and graphical solution, sensitivity analysis

1.5. Linear Programming is created through functions and graphs drawn within the first positive quadrant on a number line.[iii] Graphs that involve the remaining three quadrants are non-linear.

1.5.1. A function assigns to each number in R +1 another counterpart number in R -1.

For example f (x) = x+1 for increase by one unit on the slope of the graph. Read the equation as 'function of x is equal to x + 1. x = 0 on the number line.' Therefore for number x+ 2, linear progress assigns 3 because x+2 is represented at point 1 on the number line, and similarly for other numbers. $X+^{-3/2} = x+^{-1/2.}$ F (-3/2) = -1/2. Double its number g (x) = 2x into g(4) = 8; g(-3) = - 6.

1.5.2. When x is input and y is output, then y = x-1and y = 2x. Formula = f (x) ax^{k}.

1.5.3. The degree of the function is indicated by the highest number.

1.5.4. Slope of a linear graph is the difference between two points. $Y'2 – y^2 / 1 = y' –y_1 / 1$. The formula for the slope is $m = y_1 – y_0$ divided by $x_1=x_0$ = slope of line. Example: (4,6) and (0,7) is m = 7-6 / 0-4 = -1/4.

1.5.5. If the domain is limited, it cannot be divided by zero or by square root. (a, infinity) = {x sigma R' = x > a}, (- sigma + sigma) = R^1.

1.5.6. Places where the function increases or decreases are important as those points represent change in the direction of the graph and equation.

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1.5.7. Examples of change:

a) Function f has a minimum at x_0

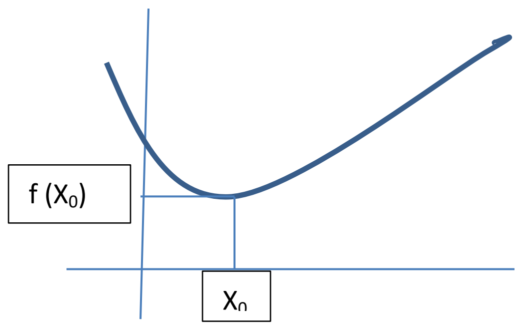


Figure 5

b) Function f_1 (x) = 3 x 4 and (0, 0) is global minimum.

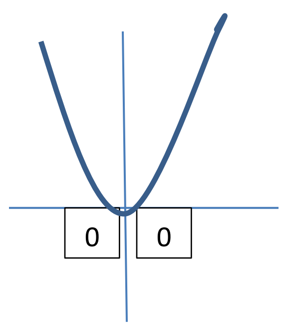


Figure 6

c) $F_3(x) = -10x^2$

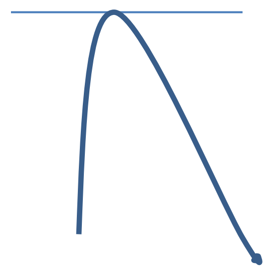


Figure 7

d) 2 of x and 3 of y

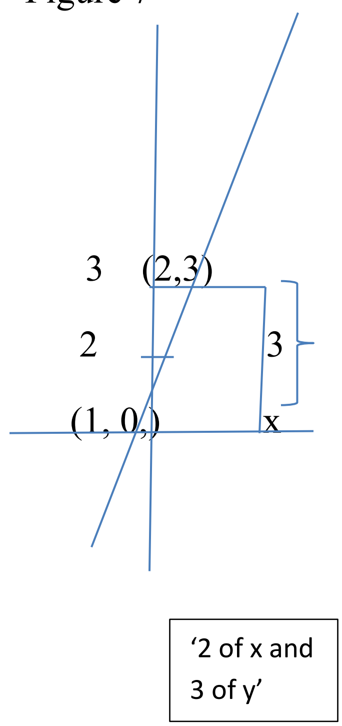


Figure 8

Source : Adapted from Lecture Notes of M. Phil (Law and Governance) and Self- study Material for MS 8 and MS 95 Research Methodology for Management Decisions, for MBA (Finance) IGNOU, and other readings.

1.6. Chapter 6 Linear Graph or Equation of a line:

1.6.1. Linear Graph or Equation of a line: A line of slope 'm' intercept 'y' (0, b) and has an equation y = mx +b.

Y= mx + b OR y-b / x-0 = m.

OR Y-b = mx.

OR Y = mx + b.

F(x) = mx + b

1.6.2. Example equation y = mx+ b. Slope (x 1 y,) and (x_2,y_2)

Y_2-y / $x_2 - x_1$ = (mx_2 + b) – (mx 1 + b) / x_2 – x1 = m ($x_2 - x_1$) / $x_2 - x_1$ = m is the slope.

1.6.3. Questions:

a) If given points are x_0, y_0 and a generic point is (x, y), compute the slope.

Answer. M = Y - y_0 / x-x_0 Then substitute this value in formula Y = mx (y_0 –mx_0)

Slope = measure of change in one unit example centigrade degree = x Fahrenheit = y $0^0 = 32^0$ C = F. 100^0C = 212^0 F. Equation for (0, 32) (100, 212) Slope = 212 – 32 / 100 – 0 = 180 / 100 = 9 / 5 Means that 1^0C = 90/5 F.

Inverse of both: F = 9/5C +32 and C = 5 / 9 (F-32)

OR

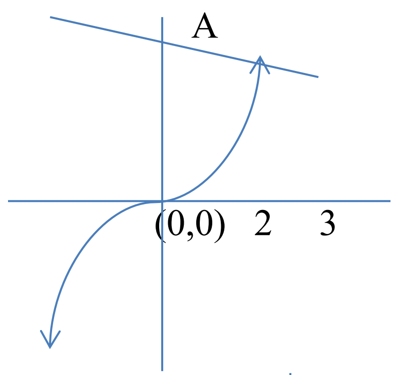
Use slope 9/5 and (0,32) to express linear equation : y-32 / x-0 = 9/5 or y = 9/5x +32.

This is marginal cost, marginal utility, and marginal product.

1.6.4. Graph with a non- linear slope:

In a linear graph slope change varies at the same rate at each point. But in a non-linear slope the change varies differently at every point.

Figure 9 Tangent line (x axis) A Non tangent line to graph of x^3



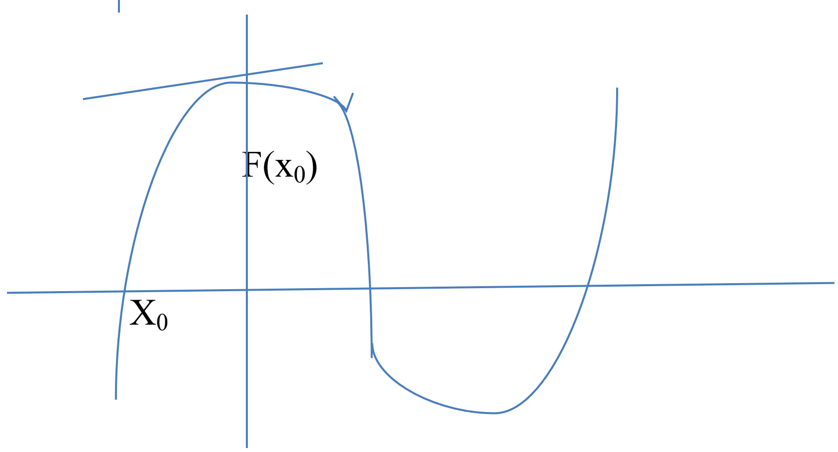


Figure 10 A rough graph of a non- linear function $f'(x_0)$ or $df / dx (x_0)$

1.6.5. Question on Studying the non-linear function

y = f (x) at point (x_0,f (x_0) in graph of Figure 2, to measure the steepness or rate of change of 'f' when x = x_0

Answer. Steps to be taken for reaching a solution:

a) Draw a tangent line at x_0.
b) The tangent is very closely approximating graph of 'f' around (x_0, f (x_0)) and is therefore, a good proxy of the graph.
c) Its slope is linear and can be measured by formula m= $y_1 – y_0$ / $x_1 – x_0$.
d) Tangent line approximations are used in daily life by architects who assume the ground to be flat instead of spherical. Approximations of tangent are exact for 10 to 20 decimal places.
e) Use derivative for measuring tangent f at x_0
f) F' (x0) or df / dx (x0) because the slope is the change in 'f' divided by the change in x, or delta f / delta x. Delta = change.

1.6.6. Analytical definition of derivative for use in economic analysis:

a) Precisely define tangent line to graph of 'f' by using the limiting process and calling part of the tangent between two points as 'secant line'.
b) Assume 'h_i to be any small number and take two points on graph (x_0, f (x_0)') and (x0 +h1, f (x0 +h1) . Draw a line L_1 joining these points
c) Choose h_2 closer to zero than h_1, and draw L_2 joining (x_0, f (x_0)) and ($x_0 + h_2$, f ($x_0 + h_2$)
d) Continue in this way choosing h_n which converging monotonically to 0. For each h_n draw a line L_n through two points of (x_0, f (x_0)) and (x_0 +h_n', f (x_0 +h_n). Lines 'Ln'

geometrically nears to the tangent line to the graph of ‘f’ at (x_0, f (x_n)).

e) Its slope is f (x_0 +h_n) – f (x0) / (x_0 +h_n) – x_0 =
f (x_0 +h_n) – f (x_0) / h_n

f) The slope of the tangent is the limit of this process as h_n converges to 0.

1.6.7. Definition: Let (x_0 , f (x0)[i]) be a point on graph y = f (x) Derivative f at x_0 written as f'(x_0) or df / dx (x_0) or dy / dx (x0) , is the slope of the tangent to graph f at (x_0, f (x_0)).
Analytically, f' (x_0) = limit h toward 0 f (x_0 +h) – f (x_0) / h .

1.6.8. Example: Tangent to graph f (x) = x_2 at x_0 = 3

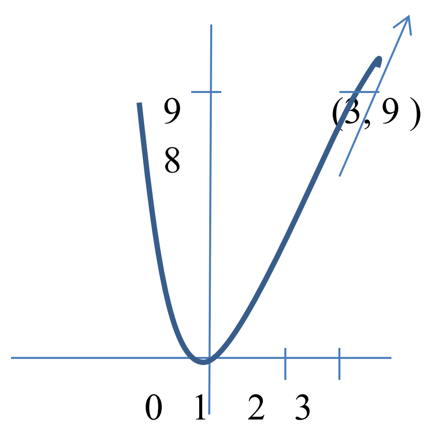


Figure 11

Analytically, f' (x0) = limit h toward 0 f (x_0 +h) – f (x_0) / h. Here, derivative of the simplest non-linear function f(x) = x2 is shown at x_0 = 3. F' (3) is larger than 1, because graph at (3, 9) is very steep as indicated by the tangent arrow line.

1.6.9. For a sequence of h_n moving towards 0,

h_n = 0.1, 0.01, 0.001, …..,$(0.1)^n$ … As h_n nears infinity at zero, quotient is nearer to 6.

h_n	X_0+h_n	$F(x_0 + h_n)$	$F(x_0+h_n)$ – f (x0) divided by h_n
0.1	3.1	9.61	6.1
0.01	3.01	9.0601	6.01
0.001	3.001	9.006001	6.001
0.0001	3.0001	9.00060001	6.0001

Also refer to NCERT Mathematic text book for classes 5 to 7 on Real numbers available here on www.10x10learning.com

Chapter 7 Regression

1.7. Regression is used to get a correlation between two or more variables in complex data. It is an approximation that is nearest to the exact functional relationship between two or more variables. Linear variables measure through mean, mode, and normal distribution. Complex variables are measured by regression.

1.7.1. For data having multiple variables regression method to create a Regression Equation is used to know the dependence of a random variable Y on quantity X which is another variable but is not random.

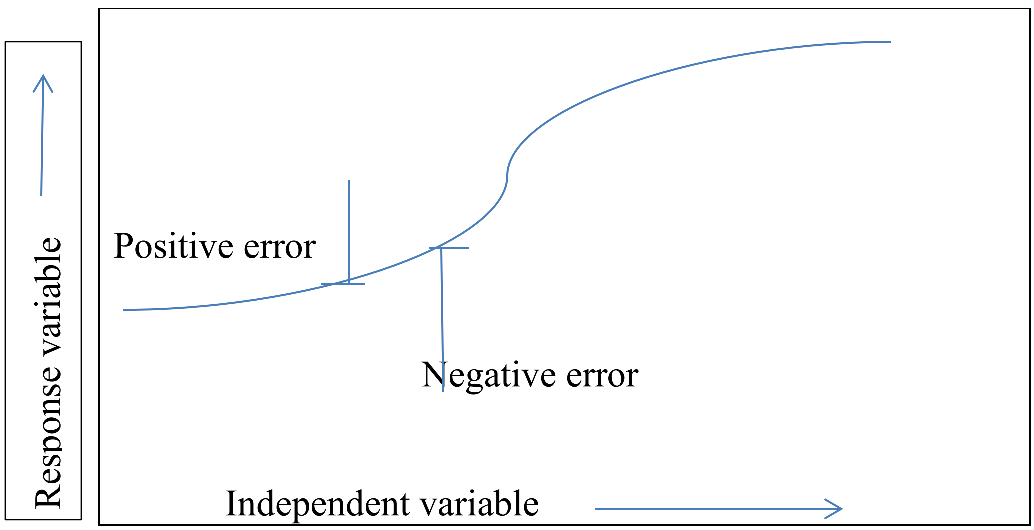
1.7.2. Steps:

a) First identify the independent variables and dependent variable.
b) Secondly, guess whether their relationship is linear, quadratic or cyclic. This helps in plotting the graph.
c) Thirdly, Estimate the parameters. It could be the fixed values form actual data ($b_0 + b_1$) as in $Y = B_0 + B_1 x + E$. All the three elements in the equation remain unknown but fixed by actual data. When examining all possible occurrence of Y and X, base values are taken from the data and represented as 'Y hat' = $b_0 + b_1X$. This will create the mathematical model.
d) Fourth step is to conduct an error analysis to see if the model fits into the actual data. (See the least square criterion graph below).
e) Fifth and last step is to see if the mathematical model is the best Least Square fit. If not repeat steps from 2 to 4 by selecting another random variable from the multiple variables.

Source : Adapted from Lecture Notes of M. Phil (Law and Governance) and Self- study Material for MS 8 and MS 95 Research Methodology for Management Decisions, for MBA (Finance) IGNOU, and other readings.

1.8. Least Square Criterion

1.8.1. Least Square criterion is also called 'Fitting a straight Line'. It has an independent variable and a least square variable. The fitted function attempts to minimize the sum of squares of errors. This is because the fitted line is the result of balancing the positive and the negative errors. This enables the error to be as small as possible. The most common criterion in the determination of model parameters is to minimize the sum of squares of errors.



1. The least square method results in linear equations for solution of parameters which are easy to solve.
2. It is a simple method.
3. It easily results in estimates of quality of fit and intervals of confidence of predicted values.
4. The sum of squares of the deviations from the true line is

$$S = \text{sigma}^{n}_{I=1} \; e^{2}_{i} = \text{sigma n} \; (Y_i - B_0 - B_1 X_i)^2$$

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1.8.2. Illustrative examples of use of statistical techniques for management decision making:

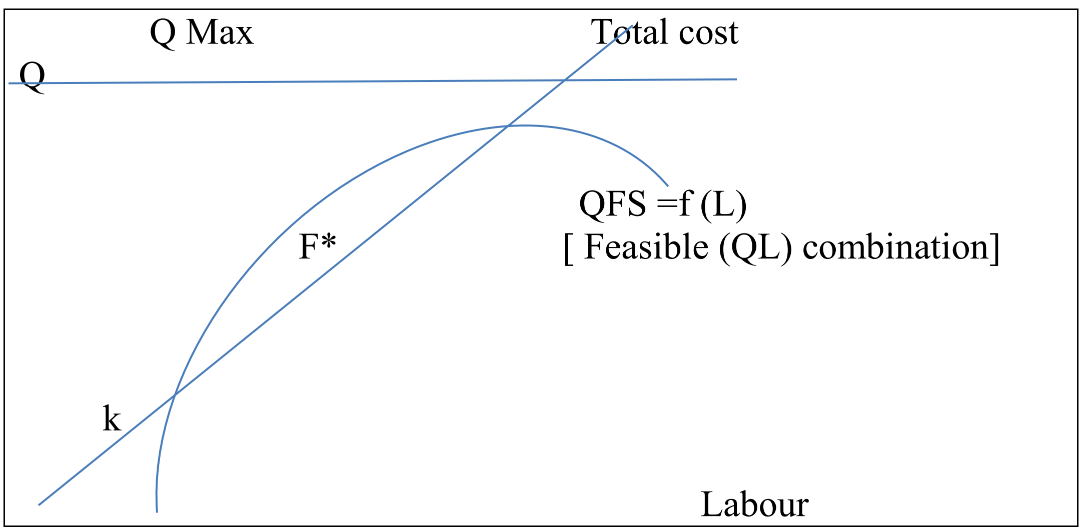
1. The Beaumol's model assumes that a firm attempts to maximise total revenue subject to a minimum profit constraint.

   Output Q = is a function of Capital (k) and Labour (L).

   Cobb-Douglas function is $Q = A\, k^{x}\, L^{beta}$

   Diminishing returns described by Marginal Productivity (MP) of either of the input factors .

   eg. MP_{L} = delta Output Q / delta labour L = Beta $A\, K^{x}\, L^{beta\ -1}$



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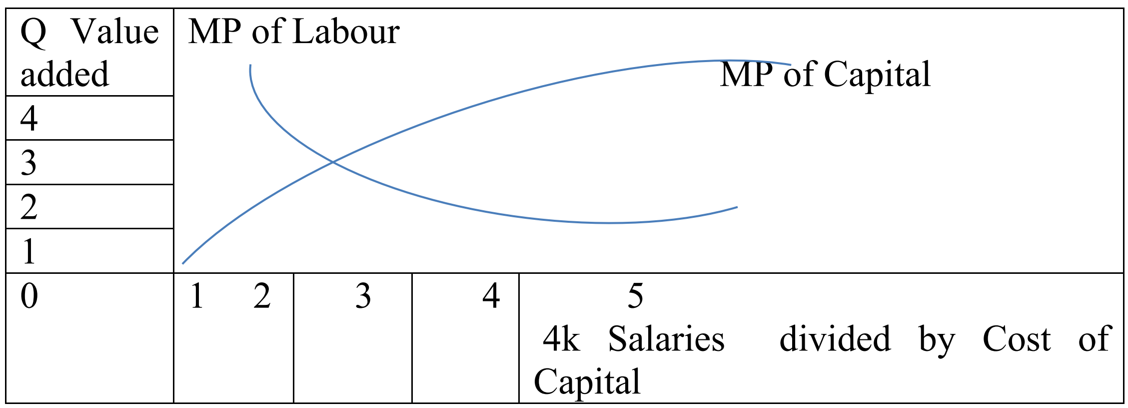
2. Production Labour: Marginal productivity (MP) of labour and capital:

MPL = MP k when LC / CC – 2.2. There is a constant elasticity of substitution of production salary cost for cost of production capital. This hypothesis has been substantiated and holds true to all models of Cobb – Douglas type where the exponents add up to unity.



3. Cost of Capacity to Produce: Output is measured by annual value added. Labour is measured by annual salaries. The interdependency between Output per capital unit and labour intensity is supported by a correlation measure of +0.55 (1% level of significance). Output per labour unit is positively influenced by the capital intensity, but the marginal productivity of production capital decreases continuously. This marginal productivity of capital nears 0 as ratio goes above 10%.

3.1 With a log linear model base, the log is + 0.84 (1%) which is higher than the measure in a linear model base of +0.55% (1%). Value of the exponent is rather small as mp nears 0.

3.2 A comparison of the two regression models similarly indicates a higher r^2 for a log linear model. Therefore, hypothesis is to be accepted Legend: Table 8.20. Output = Q. Cost of capital = k. Salary cost = L.

3.3 A comparison of the two Regression models

Regression 1	Regression 2
$Q/K = 35.7 + 0.45\ L/K$ Level of significance = 1% $r^2 = 0.306$	$Q / k = e^{1.6} + (L / K)^{0.96}$ Level of significance = 1% $r^2 = 0.708$.

3.4. Advantages of scale would imply that capital is more productive when employed on a large scale. However, the output per capital unit does not increase if total cost of capital increases. Therefore, there are no advantages of scale if more capital is invested in tools and machinery.

3.5. Output in Production capital is positively correlated with cost of capital invested in tools and machinery. Output Q is measured by annual value added and cost of capital. K is measured by depreciation and interest foregone on tools and machinery. A correlation coefficient 0.50. Hypothesis cannot be rejected on 1% significance.

3.6. Output per labour unit is positively influenced by the capital intensity, but the marginal productivity of production capital decreases continuously. This marginal productivity of capital invested in labour nears 0, as ratio goes above 10%.

Example 4

4. Cost Minimisation: Average production function indicated in the graph below, shows the isocost lines simply have a slope of -1. The isocost line indicates the ratio of price to input factors. This is because the cost of labour and cost of capital have been expressed in equal monetary terms. The graph shows how an optimal input combination depends on the isocost line. Input factors are substitutable for one another, but this elasticity depends on prices, and is defined by a measure of elasticity of substitution. (o')'

$$O' = \frac{\text{Relative change in LC / CC}}{\text{Relative change in } P_C / P_L} = \frac{\text{delta (LC / CC) / LC / CC}}{\text{delta } (P_c / P_L) / P_c/P_L}$$

Value of O' will be 0 if the two inputs are used in a fixed proportion and no substitution can take place. Alternatively, where substitutes are perfect, a small change in pries will cause radical changes in input ratio

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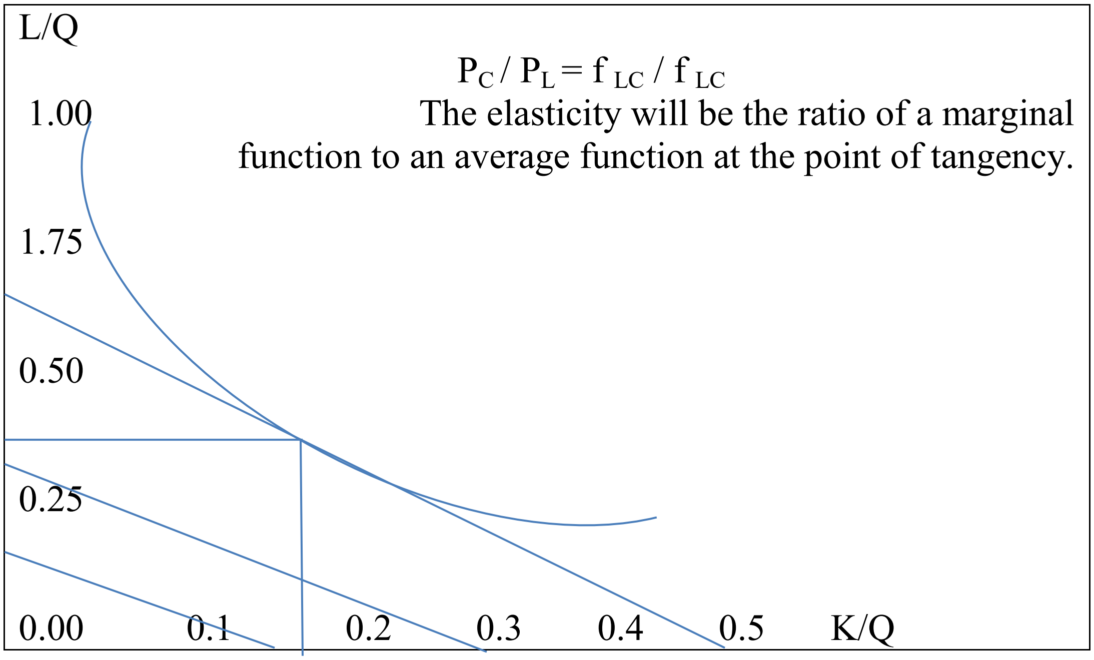

a) Derived from the graph is that the relation LC / CC = 2.2 based on price ratio PC/PL = 1/1. Annual labour cost as well as annual capital cost was measured in equal monetary flow terms.

b) This relation can also be stated as LC /CC = 2.2 P_C/P_L

Thus, the marginal = $\dfrac{\text{delta (LC / CC)}}{\text{Delta} (Pc / P_L)} = 2.2$

And the average = $\dfrac{LC / CC}{P_C / PL} = 2.2$

c) Consequently, the elasticity of substitution will be 2.2 / 2.2 = 1. This means that there exists a constant unitary elasticity of substitution in cost minimisation.

[i] From Essay ' On the Average and Scatter' by M. J. Moroney
[ii] From Essay ' On the Average and Scatter' by M. J. Moroneyi
[iii] Read the pptx on Fundamentals of Geometry and Fundamentals of Algebra under Important on the home page.